A Short Overview of Non-classical Modal Logics

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This talk is a short overview of some recent results of modal logics based on intuitionistic propositional logic. Modal logics based on IPC have a long history, starting from a 1948 paper of Fitch. Key works were written in 1970s by Ono and Fischer-Servi, 1980s by Wijesekera, and 1990s by Simpson.

In contrast to classical modal logics, $\Box$s and $\Diamond$s modalities are *not* interdefinable in non-classical modal logics. This raises the question of what should be the relation between these modalities. The two main answers are CK and IK. The logic IK is obtained by to CK axioms relating the modalities, all of which are classically equivalent to the axiom K. Even though all these axioms involve $\Diamond$s, IK proves $\Diamond$-free formulas which are not provable in CK. This was recently obtained by Das and Marin, contradicting what was previously thought as 'common-knowledge'. In contrast, if we add axioms for symmetry to CK and IK, the resulting logics coincide.

On the proof theoretical side, there has been much work on nested and labeled proof systems, where extra syntactical features relating to the Kripke semantics are added to proof systems. Such systems have been used by Girlando *et al.* to prove the decidability of IK and IS4. More recently, I developed such proof systems for non-classical variations of GL and the μ-calculus, allowing to prove their decidability.

We close with two applications. First, Mojtahedi has announced a provability logic for Heyting Arithmetic, obtained by adding to an intuitionistic version of GL extra axioms pertaining to admissibility. Much work has also been done on IGL, an intuitionistic version of GL, including multiple proof systems and decidability. Second, Artemov and Protopopescu defined IEL, containing the coreflection axiom $\varphi \to \Box\varphi$. While this axiom is not interesting classically, there are multiple logics based on it on the non-classical setting.

At last, we note that there are many other developments which we will not have time to discuss. These include, but are not limited to, topological semantics, Craig interpolation, and justification logics. Also not that not all non-classical modal logics are intuitionistic, modal logics are also being studied over subintuitionistic, intermediate, relevant, and connexive logics.