▸ LEONARDO PACHECO, *Collapsing Constructive and Intuitionistic Modal Logics.*
Institute of Science Tokyo.
E-mail: `leonardovpacheco@gmail.com`.

Das and Marin [2] recently (re)discovered that the constructive and intuitionistic variants of K do not prove the same diamond-free formulas. We show that, on the other hand, the constructive and intuitionistic variants of the modal logic KB coincide.

The constructive modal logic CK was first studied by Mendler and de Paiva [6], and the intuitionistic modal logic IK was first studied by Fischer Servi [3]. Both logics consider non-interdefinable $\Box$ and $\Diamond$ modalities. The main difference between these logics is the classically equivalent variants of the axiom K they consider. While CK only has $K_\Box := \Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$; and $K_\Diamond := \Box(\varphi \to \psi) \to (\Diamond\varphi \to \Diamond\psi)$; IK also includes the axioms $FS := (\Diamond\varphi \to \Box\psi) \to \Box(\varphi \to \psi)$; $DP := \Diamond(\varphi \vee \psi) \to \Diamond\varphi \vee \Diamond\psi$; and $N := \neg\Diamond\bot$. A semantic characterization of the axioms FS, DP, and N was recently given by de Groot, Shillito, and Clouston [5]. For more information on CK and IK, see [2, 8].

The logic IKB is obtained by adding the axioms $B_\Box := P \to \Box\Diamond P$ and $B_\Diamond := \Diamond\Box P \to P$ to IK. It was first studied by Simpson [8], who provided semantics and proved a completeness theorem for IKB. The logic CKB is similarly obtained by adding $B_\Box$ and $B_\Diamond$ to CK. Its proof theory was studied by Arisaka, Das and Straßburger [1], who provided a complete nested sequent calculus for it. As far as we are aware, there are no semantics for CKB in the literature.

We define semantics CKB and prove the completeness of CKB and IKB with respect to both our CKB semantics and the existing IKB semantics. Our proof is via canonical model arguments; the key fact is that the canonical model for CKB is an IKB-model. We then have:

THEOREM. *For all modal formula φ,* $\mathsf{CKB} \vdash \varphi$ *if and only if* $\mathsf{IKB} \vdash \varphi$.

That is, the axiom B make both constructive and intuitionistic variants of the logic KB coincide. This is quite different from what happens on variants of K, where IK to prove $\Diamond$-free formulas not provable in CK. This observation on CK and IK was recently proved by Das and Marin [2], but was also proved by Grefe in their unpublished PhD thesis [4].

It should be noted that some of the works mentioned above already pointed to the collapse of CKB and IKB. Arisaka *et al.* [1] already showed that CKB proves DP and N. Furthermore, the results of de Groot *et al.* [5] imply that the natural semantics for CKB validates FS, DP, and N.

See [7] for detailed definitions and proofs.

[1] RYUTA ARISAKA and ANUPAM DAS and LUTZ STRASSBURGER, *On Nested Sequents for Constructive Modal Logics*, ***Logical Methods in Computer Science***, vol. 11 (2015), no. 3.

[2] ANUPAM DAS and SONIA MARIN, *On Intuitionistic Diamonds (and Lack Thereof)*, ***Lecture Notes in Computer Science***, vol. 14278 (2023), pp. 283–301.

[3] GISÈLE FISCHER SERVI, *On Modal Logic with an Intuitionistic Base*, ***Studia Logica***, vol. 36 (1977), no. 3, pp. 141–149.

[4] CARSTEN GREFE, *Fischer Servi's intuitionistic modal logic and its extensions*, PhD thesis, 1999.

[5] JIM DE GROOT and IAN SHILLITO and RANALD CLOUSTON, *Constructive CK for contexts*, arXiv:2408.00262, 2024.

[6] MICHAEL MENDLER and VALERIA DE PAIVA, *Constructive* CK *for contexts*, ***Context Representation and Reasoning***, vol. 13 (2005).

[7] Leonardo Pacheco, *Collapsing Constructive and Intuitionistic Modal Logics*, arXiv:2408.16428, 2024.

[8] Alex K. Simpson, *The Proof Theory and Semantics of Intuitionistic Modal Logic*, PhD thesis, 1994.